Vladimir Voevodsky on the role of different kinds of mathematical knowledge in social practices

Andrei Rodin, University of Lorraine and Smolny Beyond Borders programme

21 mai, 2024

Plan:

Voevodsky's contributions

Vladimir Voevodsky (1966-2017) made important contributions into two different branches of mathematics:

- Algebraic Geometry;
- Foundations of Mathematics.

Voevodsky's contributions

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Vladimir also spent a significant part of his time and effort working in Applied Mathematics and, more specifically, in the Mathematical Biology. This project remained unfinished.

Voevodsky a visionary mathematician

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This is what Vladimir says about the future of mathematics in his interview to his friend and colleague Roman Mikhailov given in 2012 (my translation from Russian):

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Considering tendencies of development of mathematics as a science I realised that mathematics is at the edge of crisis, more precisely, **two crises**.

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The **second** crisis, which is less evident, concerns the fact that mathematics becomes very complex. As a consequence, once again, sooner or later mathematical papers will become too difficult for a detailed checking, and there will begin the process of accumulation of errors.

Remark

Both crises mentioned by Vladimir have societal aspects.

Interview with Roman Mikhailov, 2012, cntd

I decided to do something in order to prevent these crises. In the first case that meant to find an applied task, which would require for its solution methods of pure mathematics developed during the last years or at least during the last decades.

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Finally, I chose — as I now understand wrongly — the problem of reconstruction of history of populations [of living organisms] on the basis of their present genetic constitution. I worked on this problem about two years and finally realised in 2009 that everything that I invented was useless.

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During the same time I was looking for an approach to the problem of accumulation of errors in works of pure mathematics. It was clear [to me] that the only solution was to create a language, with the help of which people can write mathematical proofs checkable on computer.

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In 2005 I managed to formulate several ideas, which unexpectedly opened a possibility of new approach to one of the most important problems in the foundations of today's mathematics [= Univalent Foundations].

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3. Modern Mathematics: Modern Algebra (Galois theory, Group theory), Basic Topology, Logic (including Gödel Incompleteness theorems) and Set theory: (emerged in the late 19-20th c.);
4. Synthetic Mathematics: Representation theory, Algebraic Geometry, Homotopy theory (in particular, the Motivic Homotopy theory), Differential Topology (emerged in the 20-21th c.).

Remark:

The above four *layers* are not historical stages but different *kinds* of mathematics emerging in different times and coexisting today in some form. Compare an old city's infrastructure.

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- "Higher" Mathematics is integrated into most sciences [and engineering];
- Modern Mathematics is integrated into *some* sciences [and some technologies];
- Synthetic Mathematics is very poorly integrated (if at all).

Remark:

The social *integration* of mathematics should not be confused with its *effectiveness*.

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Unlike Eugene Wigner (1960) Vladimir does not see mathematics as a mere tool for doing science or anything else. Yet, the integration of mathematics with science, technology and practical life is, in Vladimir's view, a *sine qua non* of its future steady development.

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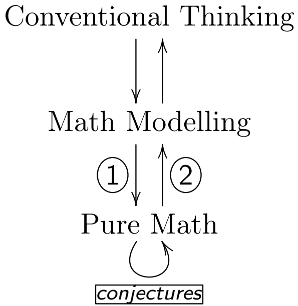
The pure mathematics grows with solving such *external* problems, which come via the mathematical modelling, and also with formulating and solving its own *internal* problems (in the form of proving mathematical *conjectures*).

Flow of Problems and Solutions



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Breakdown of arrow 2 [from pure maths to modelling] means eventually no salary for mathematicians. [Consider the poor situation of Russian Academy in the 1990s]

Breakdown of arrow 1 [from modelling to pure maths] means eventually no new ideas in mathematics. [Consider Vladimir's scientific background: he was not a typical mathematical prodigy, he never attended a special mathematical school.]

What is most important for mathematics in the near future? (Wuhan U, Dec. 2003)

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We discovered very fundamental classes of new objects including categories, sheaves, cohomology, simplicial sets. They may turn out to be as important in science as algebraic groups. But presently we don't use them for solving problems outside the Pure Mathematics.

Why? (Wuhan)

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Another reason concerns the current scientific policies. In order to apply an abstract mathematical theory to a concrete practical problem one needs, first of all, to generalise this problem and abstract away the intuition associated with this problem. But the current funding policies favour rather fast solutions of concrete practical problems such as, for example, designing "the billion dollar drug".

What can and should be done?

In order to apply mathematics to a given problem outside mathematics one should begin with the *opposite* move. Instead of trying to concentrate on future applications of a mathematical theory to the real life, one should *abstract* yourself from the real life and look at the given problem as a formal game or puzzle. This is a reason why new mathematics too often strikes one, wrongly, as what moves away from real-world problems.

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So the only reasonable policy in mathematical research and in science in general is to support one's curiosity and one's sense of beauty in science.

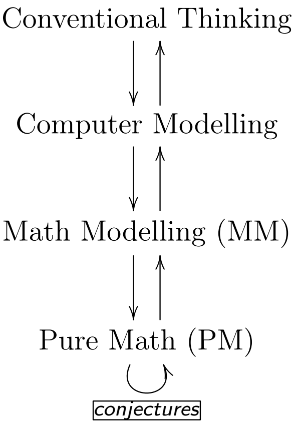
[Remark]

Vladimir does not contest the power of mathematical abstraction but rather insists on the importance of keeping in mind the "real-world problems" while doing abstract mathematics. In that way, according to Vladimir, mathematics can be most effective.

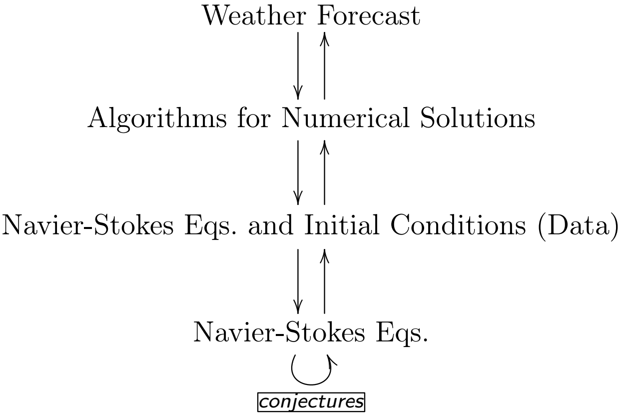
What can and should be done? (Bangalore)

To change the existing pattern of using computer technologies in the mathematical modelling.

The existing pattern



[Example] (mine)



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This particularly affects today's abstract mathematics. Problems, which pass through the filter, are formulated in the old-style language of variables and analytic functions, while the language of today's abstract mathematics is the Set theory.
[Notice the timing: this has been said by Vladimir back in 2003 before he came to the idea of building new Univalent foundations of mathematics!]

[Modifying the existing pattern]

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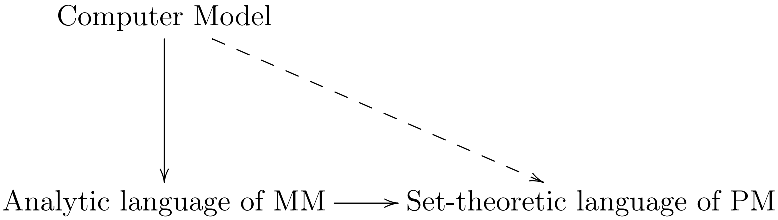
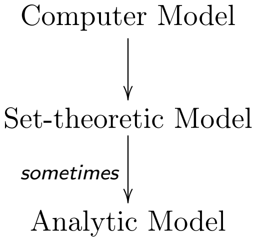
- to conceive of physical reality in set-theoretic terms directly (see below) and
- to translate today's abstract mathematics into computable terms (in an appropriate sense): Univalent Foundations.

Double Translation of Problems



The new pattern



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In order to implement the above new scheme we need, in particular, to reformulate fundamental and applied scientific theories in the language of today's abstract mathematics, viz., the set-theoretic language.

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For this end we need to specify for each theory a notion of basic *unit* and then consider sets of such units.

Sciences and Their [Ontological] Units

Science	**Unit**
Population Biology and Demography	*Individual Organisms*
Financial Mathematics	Companies
Political Science	Voters
Particles Physics	Particles
Population Genetics	Genes
Future Theoretical Chemistry, which will be able to account for individual molecules	Molecules

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- Formal ontologies (Barry Smith et al. since 1990s): useful in CS but not in science education, nor in communicating new scientific results.

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Stone's strong thesis concerning the nature of mathematics goes along with the idea of building mathematics on the Set theory, and reflects the fact that the set-based mathematics is wholly detached from its applications at the foundational level.

[Set theory in Science]

Traditional arithmetic, traditional Euclid-style geometry, classical Calculus, Riemannian geometry (to name a few) apply in sciences and technologies via modelling and supporting basic epistemic and technical procedures of empirical science such as *measurement* (including *counting*), *observation* (including astronomical observations by naked eye) and *experimental design* (think of Galileo's experiments) at the very elementary level of these theories.

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In other words, foundations of these mathematical theories make part of foundations of those scientific theories where they apply. Cf. Newton's notion of *philosophiae naturalis principia mathematica*.

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Set theory may serve as a mathematical basis of formal ontology but it does not support anything like the aforementioned epistemic and technical procedures, which allow us to acquire empirical knowledge about our environment, and which bridge mathematics with our scientific theories and our technologies.

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Lawvere 1970: a set theory . . . should apply not only to *abstract* sets divorced from time, space, ring of definition, etc., but also to more general sets, which do in fact develop along such parameters. [Lawere points here to the Topos theory that is a branch of Category theory; this is not a set theory in the usual sense of the term.]

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There is a large body of works aiming at applications of Category theory (flat and higher categories) in theoretical Physics: Lawvere, Baez, Schreiber (QFT) et al. Interestingly, Vladimir did not follow this path but looked for applications of mathematics in empirically-oriented (data-driven) research.

[Univalent Foundations as a new mathematical foundation of science?]

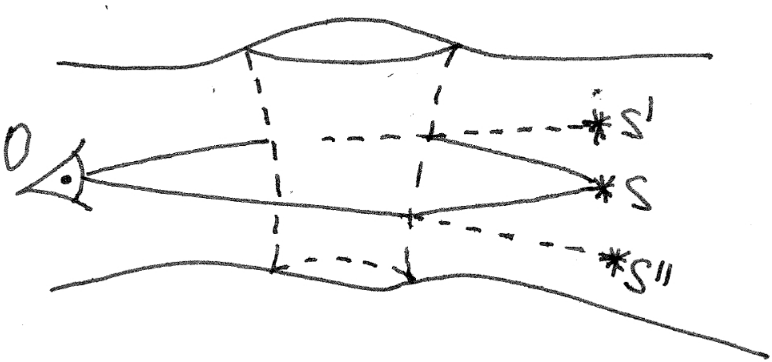

Рис.: non-homotopic paths

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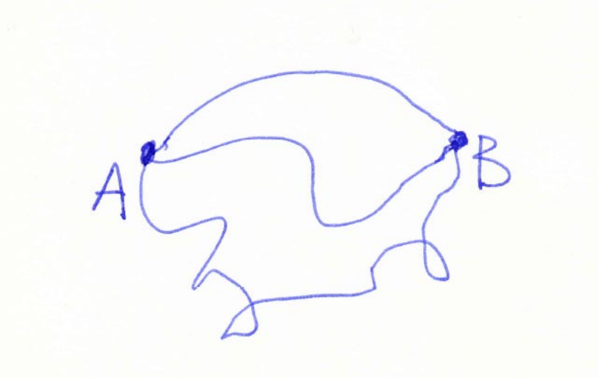

Рис.: quantum paths

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Topological Data Analysis (TDA) is a powerful method of data analysis and data visualisation based on the PHT that applies today across many theoretical and practical fields involving data analysis: medicine, finances, demography, etc.

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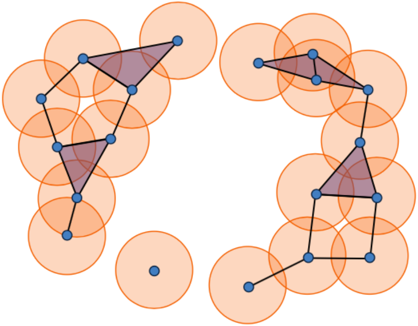
PHT has a strong backward effect on pure mathematics: Morse theory, etc.

Topological Data Analysis

TDA has direct physical interpretation. But principles of PHT/TDA are very intuitive and can be easily explained to anyone.



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"Abstract nonsense" (Category theory) can be interpreted in "concrete" spatio-temporal terms and possibly serve as a mathematical language of Science (Lawvere). The concepts of *path* and *homotopy* are just as intuitive as Euclid's concepts of *line* and *surface* albeit they frame our pre-mathematical spatio-temporal experience and our experimental practices differently.

Conclusion 2

The vital connection between pure mathematics and its applications essentially involves the foundational level of mathematics. Set-theoretic foundations of mathematics do not properly support this connection.

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This is why mathematics needs new foundations that can perform this important task without limiting the power of mathematical abstraction. Univalent Foundations and their later modifications are interesting candidates.

Conclusion 3

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The controversial outcomes of the *New Maths* reform should be taken seriously. Traditional approaches such as Euclid-style ruler-and-compass geometry should continue to be a part of elementary mathematical curricula. But new mathematical concepts, ideas and styles should also find their way to all students of mathematics at all stages of eduction.

Conclusion 3 (cntd.)

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Today's students of mathematics should get a sense of history of the subject. It is important to learn that mathematics can be developed on different foundations. But learning mathematics should not mimic the global historical development of mathematics and should not stop long before a student gets a glimpse of the state of the art in today's mathematics.

Andrei Rodin, *Voevodsky's unfinished project: Filling the gap between pure and applied mathematics*

BioSystems, vol. 204:104391 (June 2021)
DOI: 10.1016/j.biosystems.2021.104391

arXiv:2012.01150

THANK YOU!